

**REDUCTION OF METHODOLOGICAL ERRORS  
IN DETERMINING THE TEMPERATURE OF  
METALS BY TWO-COLOR PYROMETERS**

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*Two-color methods of optical control of the thermodynamic temperature of metals subjected to thermal treatment are analyzed. A technique for taking into account the nonmonochromaticity of the photoreception path is suggested and an analysis of methodological errors of two-color pyrometers with wide spectral ranges is performed by numerical simulation.*

**Bichromatic Methods.** The most widespread of bichromatic methods is the method of spectral ratio. It is based on determination of the ratio of spectral energetic brightnesses  $L(\lambda_1, T)$  and  $L(\lambda_2, T)$  of the radiation arriving from a heated object at a photoreceiver [1, 2]. Determining  $L(\lambda_1, T)/L(\lambda_2, T)$ , with the ratio  $\epsilon_2/\epsilon_1$  known beforehand, we can find the thermodynamic temperature:

$$\frac{1}{T} = \frac{1}{c_2} \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \left[ \ln \left( \frac{L(\lambda_1, T) \lambda_1^5}{L(\lambda_2, T) \lambda_2^5} \right) + \ln \left( \frac{\epsilon_2}{\epsilon_1} \right) \right]. \tag{1}$$

Since the method does not require knowledge of the absolute values of  $L(\lambda_1, T)$  and  $L(\lambda_2, T)$ , the results of determining  $T$  within some limits is independent of the distance to the object and of the variations of uncontrolled parameters (contamination of an objective, dustiness of the medium, etc.).

The methodological error of the bathochromic method of spectral ratio is equal to [1, 2]

$$\delta_{sp,r} = \frac{1}{c_2} \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \ln \frac{\epsilon_2}{\epsilon_1}. \tag{2}$$

To eliminate this error, one has to know exactly the ratio of the radiating capacities of the material  $\epsilon_2/\epsilon_1$ , which under real working conditions is not always possible. To raise the signal/noise ratio, it is desirable to use rather wide spectral ranges. Here real pyrometric signals will be proportional to

$$U_i(T) \sim \int_0^\infty \tau_{if}(\lambda) \epsilon(\lambda, T) S(\lambda) L(\lambda, T) d\lambda = \int_0^\infty \epsilon(\lambda, T) S_i(\lambda) L(\lambda, T) d\lambda. \tag{3}$$

In using nonmonochromatic spectral ranges, there is sense in talking not about the monochromatic radiating capacity of the body surface at a certain wavelength  $\lambda$ , but rather about the effective radiating capacity  $\epsilon$  (effective emissivity) which, for the  $i$ th spectral range, can be determined as follows:

$$\epsilon_i = \frac{U_i}{U_i^{bb}} = \frac{\int \epsilon(\lambda, T) L(\lambda, T) S_i(\lambda) d\lambda}{\int L(\lambda, T) S_i(\lambda) d\lambda}. \tag{4}$$

This technique, referred to in the literature as a direct comparison with a standard [3, 4], is based on the equality

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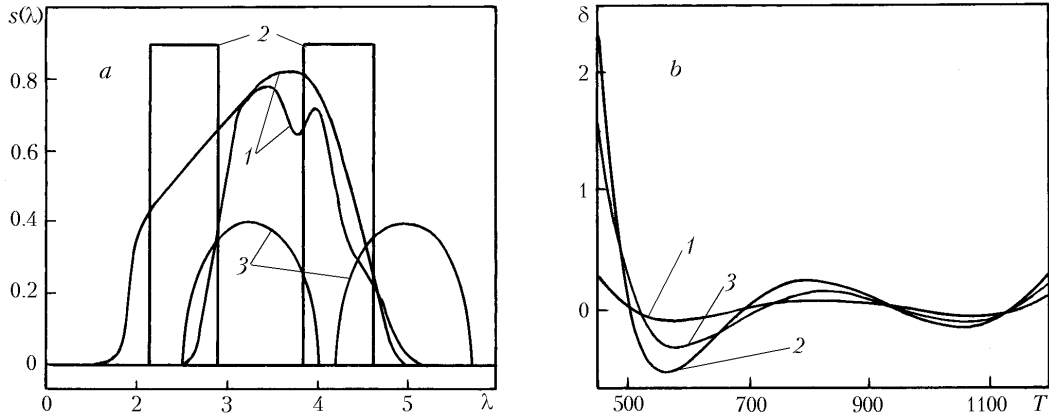


Fig. 1. Simulated spectral sensitivities of a photoreceiving device with different speeds of change in  $s(\lambda)$  (a) and the resulting corresponding temperature dependences of the relative deviation of approximation from the true values of  $U_1^{bb}/U_2^{bb}$  (b).  $\lambda$ ,  $\mu\text{m}$ ;  $\delta$ , %;  $T$ , K.

$$\varepsilon_i = \varepsilon_i^{\text{st}} \frac{U_i}{U_i^{\text{st}}}. \quad (5)$$

As a standard, one can use a cavity of cylindrical or conical shape made directly in the sample under control. At the area of the exit hole that is greatly smaller than the area of the inner surface of the cavity, its emissivity is close to unity. Thus, by measuring pyrometric signals from the plane portion of the controlled sample surface and from the cavity, it is possible to determine  $\varepsilon_i$  with the aid of Eq. (5). Such an approach is quite acceptable in readjusting the line to a new kind of product, since only one billet of an article is expended for determining the value of  $\varepsilon_i$ .

In the case of wide spectral intervals, one fails to obtain a simple analytical expression for  $T$  which would be similar to Eq. (1). Therefore, in describing the proposed algorithm for computing  $T$  and investigating its methodological errors, we shall avail ourselves of numerical methods.

**Nonmonochromatic Method of Spectral Ratio.** In measuring the temperature by a nonmonochromatic two-color pyrometer, at the exit from its detector block the pyrometric signals  $U_1(T)$  and  $U_2(T)$  will be formed, which are described by Eq. (3). In order to determine the thermodynamic temperature  $T$ , it is necessary to preliminarily know the dependence of the ratio of effective emissivities on  $T$ :

$$\frac{\varepsilon_2}{\varepsilon_1}(T) = \frac{\varepsilon_2(T)}{\varepsilon_1(T)} = \frac{U_2}{U_2^{bb}} \frac{U_1^{bb}}{U_1} = \frac{U_2}{U_1} \frac{U_1^{bb}}{U_2^{bb}}. \quad (6)$$

Numerical simulation in the MatLab computer mathematics system has shown that the temperature dependence of the ratio of pyrometric signals for a blackbody (bb)  $U_1^{bb}/U_2^{bb}$  within the range 450–1200 K can well be approximated by a third-order polynomial:

$$\frac{U_1^{bb}}{U_2^{bb}} \approx a_3 T^3 + a_2 T^2 + a_1 T + a_0. \quad (7)$$

Figure 1b demonstrates the temperature dependence of the relative deviation of polynomial approximation (in percent) on the true values of the ratio  $U_1^{bb}/U_2^{bb}$  for different kinds of spectral sensitivity of photodetectors (given in Fig. 1a). It is seen that in the range 500–1200 K the maximum deviation from the true value of  $U_1^{bb}/U_2^{bb}$  does not exceed 1%. In order to determine the coefficients of the approximating polynomial  $a_i$ , one can measure the pyrometric signals received from the blackbody model. Having determined the ratio  $U_1^{bb}/U_2^{bb}$  for four values of temperature, we obtain a system of four equations:

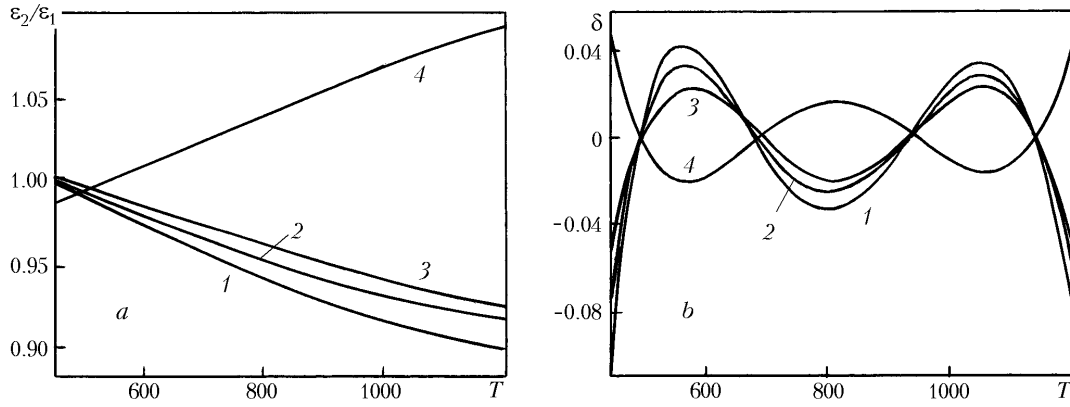


Fig. 2. Graphs of  $\frac{\varepsilon_2}{\varepsilon_1}(T)$  for tungsten (1), aluminum (2), iron (3), and alumina (4) (a) and corresponding temperature dependences of the relative deviation of polynomial approximation of the function  $\frac{\varepsilon_2}{\varepsilon_1}(T)$  from its true values (b).  $\delta$ , %;  $T$ , K.

$$\begin{aligned} a_3 T_1^3 + a_2 T_1^2 + a_1 T_1 + a_0 &= f_1, & a_3 T_2^3 + a_2 T_2^2 + a_1 T_2 + a_0 &= f_2, \\ a_3 T_3^3 + a_2 T_3^2 + a_1 T_3 + a_0 &= f_3, & a_3 T_4^3 + a_2 T_4^2 + a_1 T_4 + a_0 &= f_4, \end{aligned} \quad (8)$$

the solution of which yields  $a_i$ . The values of the coefficients of the polynomial  $a_i$  depend only on the form of the spectral sensitivity  $S(\lambda)$  used in the given pyrometer of the radiation detectors; therefore in adjusting the pyrometer, one calibration against a blackbody will be sufficient.

Since the ratio of the effective radiating capacities  $\varepsilon_2/\varepsilon_1$  is a function of temperature, a polynomial approximation can be applied to it.

$$\frac{\varepsilon_2}{\varepsilon_1}(T) \approx v_3 T^3 + v_2 T^2 + v_1 T + v_0. \quad (9)$$

Figure 2a shows the dependences of the ratio  $\varepsilon_2/\varepsilon_1$  on temperature for various materials, and Fig. 2b presents the resulting dependences of the deviation of the polynomial approximation on true values of  $\varepsilon_2/\varepsilon_1$ . It is seen that these deviations do not exceed 0.1%. In order to find the coefficients of the approximating polynomial  $v_i$ , it is necessary to obtain the ratio  $\varepsilon_2/\varepsilon_1$  for four values of the temperature of the cavity-containing sample. By measuring the ratio of pyrometric signals from the cavity  $U_1^{\text{bb}}/U_2^{\text{bb}}$  and knowing the coefficients of  $a_i$ , we can find the sample temperature  $T$  from Eq. (7). Having measured then the ratio of pyrometric signals for the sample surface  $U_2/U_1$ , we find the ratio  $\varepsilon_2/\varepsilon_1$  for the given temperature from Eq. (6). Thus, we obtain a system of equations for finding  $v_i$ :

$$\begin{aligned} v_3 T_1^3 + v_2 T_1^2 + v_1 T_1 + v_0 &= q_1, & v_3 T_2^3 + v_2 T_2^2 + v_1 T_2 + v_0 &= q_2, \\ v_3 T_3^3 + v_2 T_3^2 + v_1 T_3 + v_0 &= q_3, & v_3 T_4^3 + v_2 T_4^2 + v_1 T_4 + v_0 &= q_4. \end{aligned} \quad (10)$$

Now, for the temperature to be found we may compile the equation

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{U_1}{U_2} = \frac{U_1^{\text{bb}}}{U_2^{\text{bb}}}, \quad (11)$$

which, in using the approximation by the third-order polynomial, takes the form

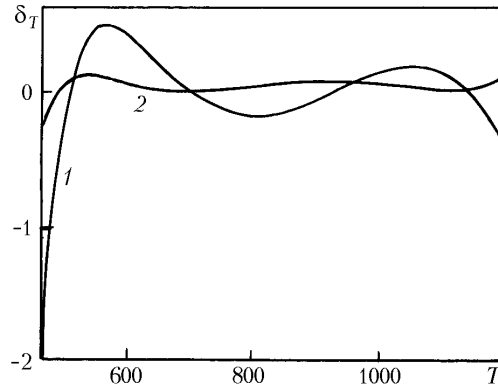


Fig. 3. Relative error in determination of the temperature of iron in approximation by third-order (1) and fourth-order (2) polynomials.  $\delta$ , %;  $T$ , K.

$$\left(v_3 T^3 + v_2 T^2 + v_1 T + v_0\right) \frac{U_1}{U_2} = a_3 T^3 + a_2 T^2 + a_1 T + a_0. \quad (12)$$

By transferring all the terms to the left and grouping the coefficients of the same powers of  $T$ , we obtain a third-degree equation for finding the temperature:

$$\left(v_3 \frac{U_1}{U_2} - a_3\right) T^3 + \left(v_2 \frac{U_1}{U_2} - a_2\right) T^2 + \left(v_1 \frac{U_1}{U_2} - a_1\right) T + \left(v_0 \frac{U_1}{U_2} - a_0\right) = 0. \quad (13)$$

The graph of the relative error in the determination of the temperature of iron is given in Fig. 3. It is seen from the figure that the methodological error in the determination of  $T$  for iron within the range 500–1200 K does not exceed 1%. The main reason for its appearance in such a technique of temperature determination is the deviation of the polynomial approximations of the functions  $U_1^{bb}/U_2^{bb}$  and  $\varepsilon_2/\varepsilon_1$  from their true values. In order to reduce these deviations, one can use approximation of these functions by a fourth-order polynomial.

The disadvantage for the proposed method is that it requires knowledge of the value of  $\varepsilon_2/\varepsilon_1$ . Therefore, under working conditions, in readjusting a line to a new kind of product, for each new material with unknown radiative characteristics one has to carry out measurements for one sample with a cavity at four values of temperature. After calibration, the values of the coefficients  $v_i$  obtained for the metal can be preserved to avoid repeated measurements when the metal is used again.

Thus, a technique for determining the thermodynamic temperature by the nonmonochromatic method of spectral ratio has been suggested. Investigation of errors has shown that the methodological error in the determination of the temperature of iron within 500–1200 K does not exceed 1%.

## NOTATION

$a_0, a_1, a_2, a_3, v_0, v_1, v_2,$  and  $v_3$ , coefficients of approximating polynomials;  $c_2 = 1.43 \cdot 10^{-2}$ , second radiation constant, m·K;  $L(\lambda, T)$ , spectral density of energetic brightness;  $T$ , thermodynamic temperature, K;  $S(\lambda)$  and  $s(\lambda)$ , absolute and relative sensitivities of the radiation detector at the wavelength  $\lambda$ ;  $U_i$ , pyrometric signal from a controlled sample in the  $i$ th spectral range, V;  $U_i^{bb}$ , pyrometric signal from a blackbody in the  $i$ th spectral range, V;  $U_i^{st}$ , pyrometric signal from a standard in the  $i$ th spectral range, V;  $\delta$ , relative deviation of the polynomial approximation of a function from its true values, %;  $\delta_T$ , relative error in determination of temperature, %;  $\delta_{sp,r}$ , methodological error in determination of temperature by the method of spectral ratio;  $\varepsilon$ , radiating capacity;  $\varepsilon_1$  and  $\varepsilon_2$ , radiating capacities in two-channel measurements;  $\varepsilon_i$ , effective radiating capacity of a sample in the  $i$ th spectral range;  $\varepsilon^{st}$ , radiating capacity of a standard;  $\lambda$ , radiation wavelength,  $\mu\text{m}$ ;  $\lambda_1$  and  $\lambda_2$ , radiation wavelengths in two-channel measurements,  $\mu\text{m}$ ;  $\tau_{if}$ ,

transmission coefficient of an optical filter in the  $i$ th spectral range;  $f_j = \frac{U_1^{bb}}{U_2^{bb}}(T_j)$ ;  $q_j = \frac{\varepsilon_1}{\varepsilon_2}(T_j)$ ;  $S_i(\lambda) = \tau_{if}S(\lambda)$ . Subscripts: sp.r, spectral ratio; f, filter;  $i$ , 1, 2, subscripts and numbers of spectral channels;  $j$ , values of temperature in simulation of the calibration process. Superscripts: bb, blackbody; st, standard.

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